Efficient Origami Construction of Orthogonal Terrains using Cross-Section Evolution

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keywords: cross sections, orthogonal terrains, time evolution

Abstract

Many algorithms and universality results exist for producing parameterized families of origami structures, but few are provably efficient, i.e. provide constructions from a paper having dimensions within a low constant factor of an optimal construction. At 5OSME, Demaine et al. (2010) presented a efficient construction for folding orthogonal mazes which is computable in polynomial time. Origamizer presented in Demaine and Tachi (2017) constructs foldings corresponding to general polyhedral surfaces, but does not provide any bound on the efficiency of the constructions. On the other hand, Treemaker from Lang (1996) produces efficient crease patterns to fold uniaxial bases, but may require exponential time to to find an efficient solution.

In this paper, we present an algorithm for efficiently producing an origami folding that corresponds to an input **orthogonal terrain** with arbitrary rational extrusion heights. A folding corresponds to an orthogonal terrain if the folding covers every point on the terrain, but no point on the folding exists above the terrain. This result improves an algorithm, Benbernou et al. (2010) also presented at 5OSME, applicable to a more general class of inputs, providing a universal construction to fold general orthogonal polyhedra, though the construction is less inefficient than our construction applied to orthogonal terrains. Our construction approach follows three steps:

1. decompose the orthogonal terrain into strips that are constant along one dimension;

2. cover the strips efficiently using rectangular strips of paper; and

3. stitch the strips together along matching boundaries.

In order to better communicate the algorithm and the final folded state produced, we also introduce a new **cross-section evolution** representation of a folded isometry: a straight line is swept across the crease pattern of a folded surface, and we keep track of how the folding of the line evolves as a cross-section of the folded surface. The propagation of the cross-section between crease pattern vertices is uniquely determined by the initial orientation of the cross-section, so the folded isometry can be constructed by sweeping the line and locally modifying the cross-section when crossing crease pattern vertices during propagation. This representation not only simplifies the description of the 3D folded isometries constructed, but also provides a simpler framework to argue that the folded state does not self intersect, by propagating planar cross-sections monotonically along a single direction. We then show that our construction's efficiency is within a small constant factor of any folding with optimal efficiency.

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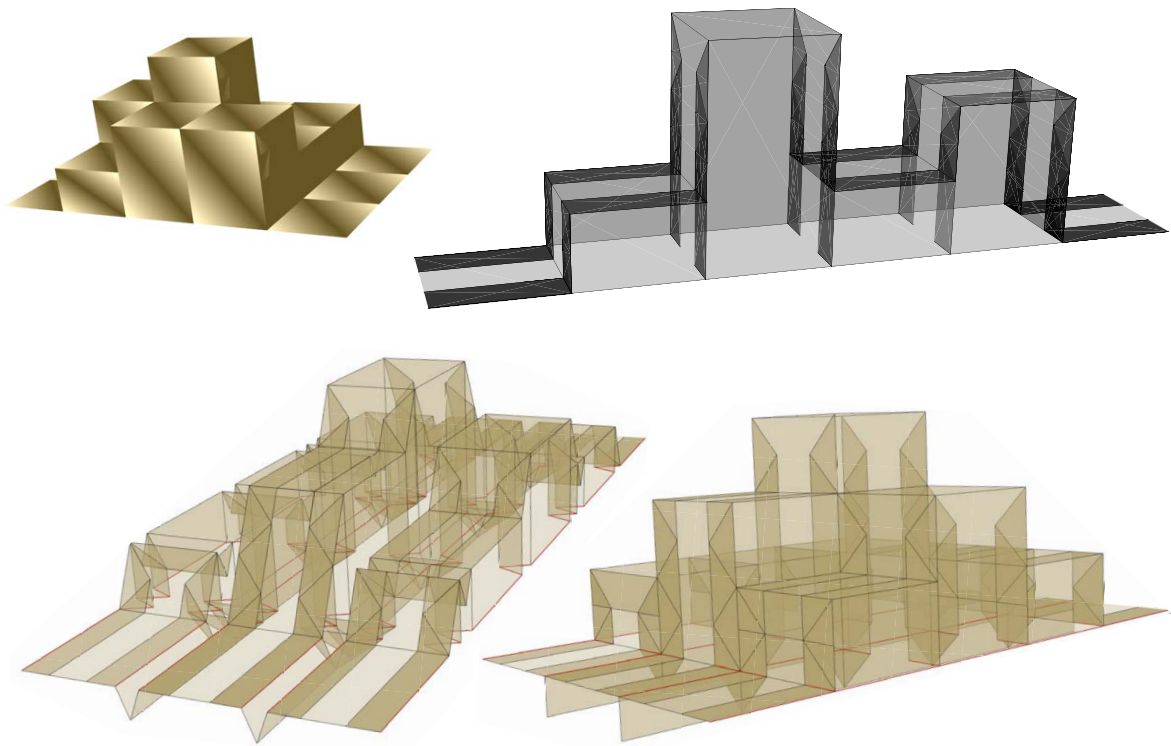
Figure 1: The construction progression a folding corresponding to an input orthogonal terrain [top left]. We split the terrain into sections that are constant along one direction, cover each strip with a rectangular strip [top right], and then recombine the sections [bottom].
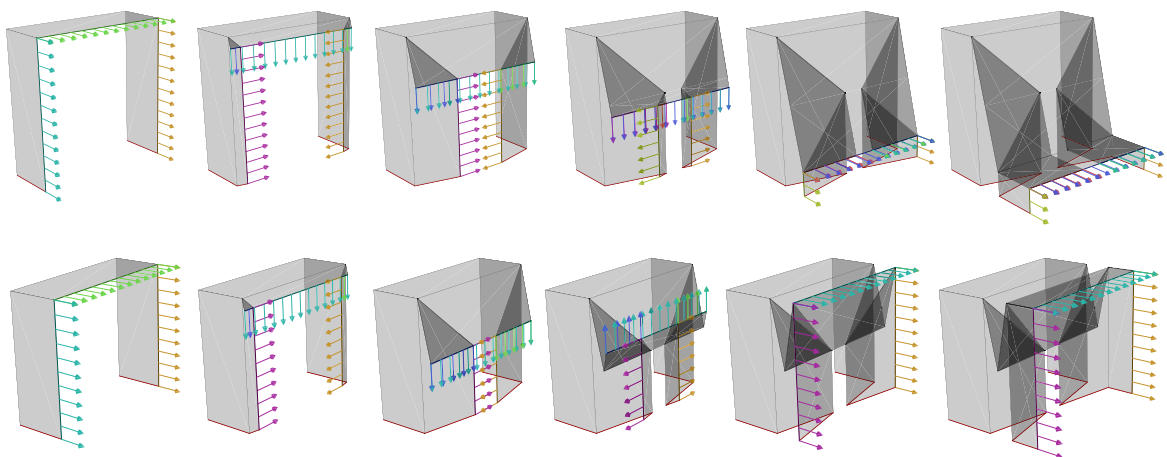

Figure 2: Snapshots of a cross-section evolution for two gadgets used to construct orthogonal terrains. The sequence on top shows a level-shifting gadget that changes the height of a section via the use of auxiliary pleats to tuck away excess paper. The sequence on the bottom shows a paper-absorbing gadget that allows adjacent sections of paper that will later be attached together to stay in sync with each other.