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FOLDING THICK MATERIALS USING AXIALLY VARYING VOLUME TRIMMING

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ABSTRACT

Many new and complex methods have been proposed for building folded structures from thick materials. Volume trimming was one of the first and simplest techniques for fabricating ideal hinges using thick materials, though has remained largely unmodified since it was proposed. This paper extends and customizes constant thickness volume trimming for layered fabrication processes. Previously, constant thickness volume trimming uniformly removed material local to each hinge, leaving layers near the hinge vulnerable to delamination. We provide a method to retain material on both sides of the fold line to create stronger hinges by varying the profile of removed material along the axis of the hinge, even for hinges with large fold angle.

INTRODUCTION

In recent years, many methods have been proposed for fabricating folded structures from thick material. Each method has its own strengths and weaknesses and provides a designer with a range of options to fit a given application. A recent survey [1] provides a good summary of existing techniques, while Figure 1 is a common depiction of how each technique differs in the plane normal to a hinge direction. This cross sectional view of thick folding is so universal, it is perhaps not so surprising that realizations of these methods commonly apply a method uniformly along the length of a hinge. In particular, the constant thickness volume trimming method uniformly removes material local to each hinge, leaving layers near the hinge vulnerable to delamination. Of course, layered manufacturing processes can easily accommodate variation in the transverse dimension. In this paper, we apply an axially varying strategy to the method of volume trimming which naturally extends to the offset crease method to allow for more robust hinge fabrication.

For shallow fold angles, axially varying volume trimming simply requires an alternation of which side material is removed from near a hinge. However, in order to retain material near the hinge for large fold angles, material must be removed from both sides of the fold plane, also requiring removal of material at the fold plane.

EXISTING THICK FOLDING TECHNIQUES

There are many existing approaches that seek to account for material thickness in folding applications, each with their own strengths and weaknesses.

A: Hinge Shift

The hinge shift strategy shifts hinges out of plane to accommodate material thickness [2]. While readily useful in creating one-dimensional foldings of thick material, this technique is harder to apply to 2D crease-pattern networks. A recent result [3] has shown cases where the hinge shift method is viable for certain internal vertices, but it is not clear if this method can be extended to more general vertices.

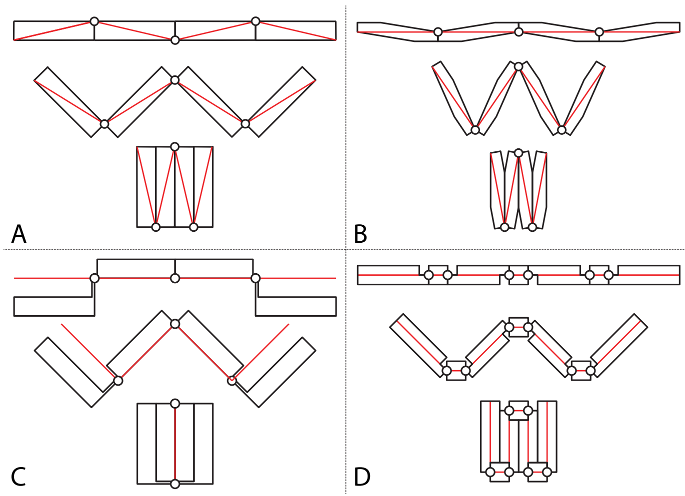

FIGURE 1. Existing thick folding techniques: (A) hinge shift, (B) volume trimming, (C) offset panel, and (D) offset crease.

B: Volume Trimming

The strategy presented in [4] trims the edges of a thickened surface to overcome many of the difficulties of the hinge shift technique. Addressing some of its deficiencies is the primary focus of this paper and will be covered in more detail in the following section.

C: Offset Panel

The offset panel technique [5] retains hinges at the folding plane, but shifts thick material away from it. While promising, fabricating such structures can be difficult requiring robust standoffs to connect thick material to hinges.

D: Offset Crease

The offset crease technique [6] compensates for thickness by splitting each crease into two. This method allows full range of motion but does add additional degrees of freedom. Because this method transforms all hinges into two hinges that fold to no more than 90°, the methods presented in this paper can be applied to the offset crease as well.

E: Rolling Contacts

The rolling contacts method [7] repurposes the toy Jacob's Ladder mechanism to construct hinges that can provide a variable shift based on a designable roll profile.

F: Strained Joint

The strained joint technique [8] induces hinges in a monolithic sheet of material by selectively removing material at the hinge. Similar to the results presented in this paper, this is a generic hinge construction technique that also utilizes cross-sectional variation along the fold axis. But unlike our method, the hinges produced by the strained joint technique necessarily distribute hinge bending over a finite area with finite curvature. Our method can theoretically achieve *ideal*, or in practice *near-ideal* hinges that behave as if the hinge is restricted to a line, leading to tighter folding motions that are easier to analyze and predict.

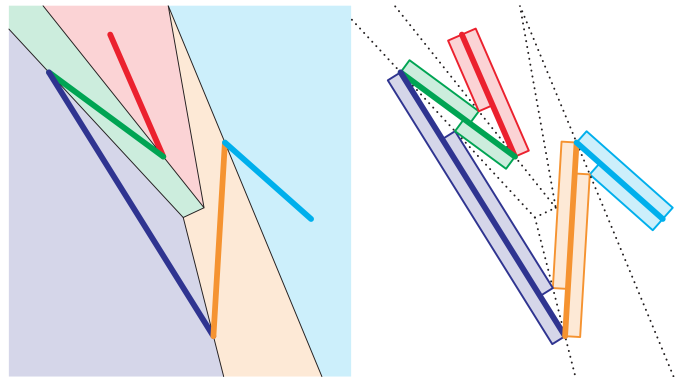

FIGURE 2. [Left] Voronoi cells of a 2D folded state. [Right] Constant width panels attached to the folding layer, contained within each Voronoi cell.

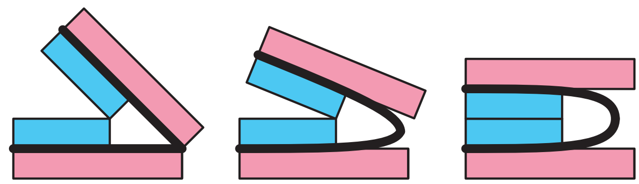

FIGURE 3. Delamination when the folding layer is not sandwiched between thick panels.

VOLUME TRIMMING

Volume trimming was introduced in [4] by Tachi and is a natural and simple approach to fabricating thick folded structures. Given a rigid-foldable folded state whose faces do not touch except at fold lines, construct the Voronoi diagram generated by the faces of the folding, and allow a 'thickened' version of a face be a subset of the Voronoi cell containing the face; see the left of Figure 2. However, fabricating such surfaces maximally is almost never done in practice because the bisectors between faces form slanted sides that are hard to fabricate.

We would prefer to build folded structures using standard sheet material of constant thickness that can be cut using simple

2-axis cutting machines, like milling machines, laser cutters, or water jets. So instead of fabricating these so called 'tapered panels', Tachi suggests thickening each side of a face with the largest prism of a constant thickness contained in the Voronoi cell; see the right of Figure 2. Fabricating each prism is then trivial using 2-axis cutters. The thick folding can then be assembled by attaching the thick prisms onto thin, flexible material that acts as the folding plane. The thick panels provide structural stiffness, while the folding plane provides hinge compliance.

This method works well if the thick material can be strongly adhered to the thin folding surface. In practice however, because the folding surface is not sandwiched on both sides by structural material near a hinge, delamination often occurs as shown in Figure 3. One way to solve this problem would be to refrain from trimming thick material on one inner side of the crease. Of course, because some material must be cut away to accommodate the folding, we cannot refrain from trimming thick material on both inner sides of the crease everywhere along the hinge. But we *can* vary how we trim material along the length of the crease, which we discuss in the following sections.

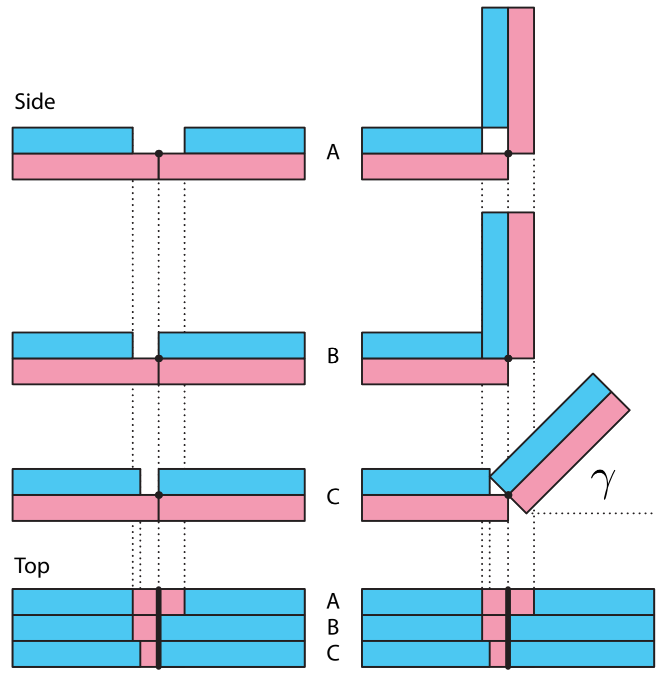

FIGURE 4. For small fold angles ($0^\circ \leq \gamma \leq 90^\circ$), volume trimming can easily be improved to remove less material. The top figures show axial cross-sections while the bottom figures show the hinges when seen from above.

AXIAL CROSS SECTION VARIATION

Let us take a closer look at the cross section for a constant thickness volume trimming hinge with layer thickness t, like the one shown in Figure 4 (A). An immediate improvement can be made to this trimming profile by removing material only on one inner side of the hinge, as in Figure 4 (B) and (C). For fold angles γ between 0 and 90°, we can leave one inner side of the hinge completely untrimmed, only trimming enough material to achieve the desired fold angle, specifically trimming at distance $t \cot \gamma$ from the fold line. By sandwiching the folding layer between the thick panels all the way to the fold line, the folding layer will be constrained from delamination on that side. If a side sandwiches the folding layer at the fold line between thick panels in this way, we will call that side *contained*.

If we want to achieve containment on both sides of the fold line, we simply alternate the inner side that we trim, along the crease direction. This alternation will form 'teeth' on the inside of the hinge along either side; see Figure 5. While zero alternation was susceptible to delamination via translation, a single alternation as shown on the left is still susceptible to delamination via rotation of the layers about the alternation point. Two alternations as shown in the middle are sufficient to suppress the rotation, though more alternations can also be used, as shown on the right.

For larger fold angles γ between 90° and 180°, the original construction still trims both inner sides as shown in Figure 6 (A). Figure 7 describes the offset geometry needed for various cross-sections for large angle strategies. Again, we can get away with trimming less as in Figure 6 (B), but even so, neither side will be contained. One solution would be to remove not just material

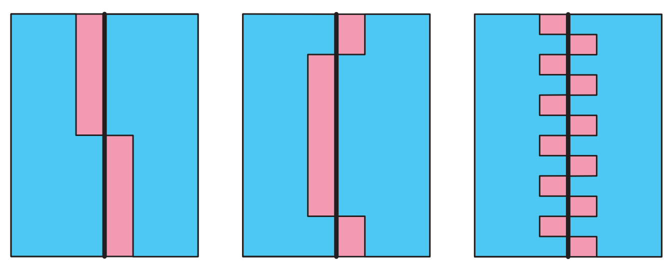

FIGURE 5. Top view of piecewise constant axial variation in volume trimming for one, two, and many alterations, from left to right respectively.

from the inner panels of the fold, but also from the outer panels as shown in Figure 6 (C). However, removing material in this way would make the material disconnected at the hinge at that point. Thus even if we alternated between (C) and (C') along the hinge, the hinge would cease to be connected.

To fix this connectivity issue, we could alternate between (B) and (C) cross-sections so that the hinge remains connected while allowing crease containment. This design may satisfy our criteria, but is somewhat lacking; this approach removes a significant

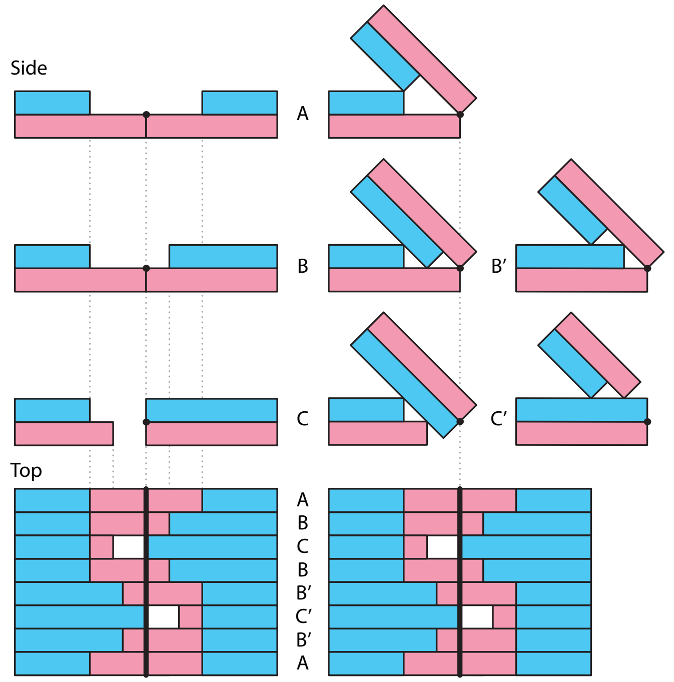

FIGURE 6. For large fold angles ($90^\circ \leq \gamma \leq 180^\circ$), naive volume trimming must be modified to achieve containment. The top figures show axial cross-sections while the bottom figures show hinges when seen from above.

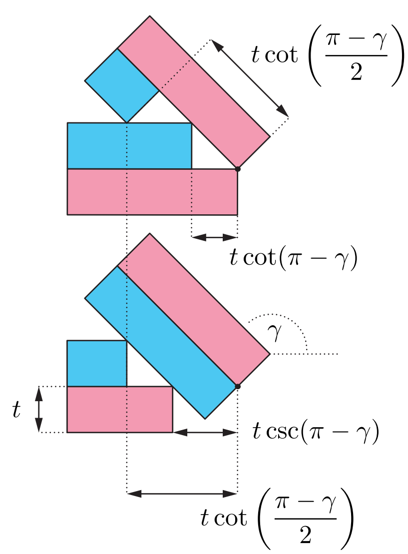

FIGURE 7. Distances for various trimming offsets when applying volume trimming to large fold angles ($90^\circ \leq \gamma \leq 180^\circ$).

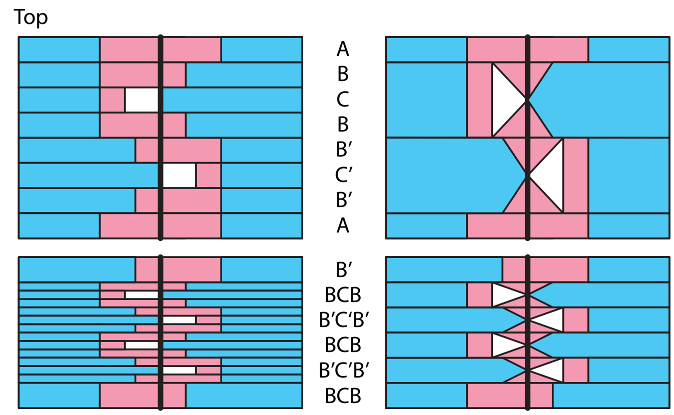

FIGURE 8. Transforming the piecewise constant axial variation on the left to a piecewise linear, diagonal tooth profile shown on the right, for two choices of tooth width.

amount of the folding layer along the hinge: the precise location where the strength of the folding layer is most significant. So far, we have only exploited cross-sectional variation along the hinge axis in a piecewise constant manner. Allowing linear variation of the cross section between (B) and (C) solves the problem, as shown in Figure 8. Combining cross-sections (B)(C)(B) into a diagonal tooth, retains folding layer material along the fold line while keeping connectivity and allowing containment.

SINGLE VERTEX SOFTWARE

The previous section dealt with cross-sectional variation strategies along a fabricated hinge. This section describes a web application[1] that was developed to facilitate generating and fabricating some simple examples of thick foldings using this method. A screenshot of the interface is shown in Figure 9.

Using the interface, one can access the full range of flat-foldable four-crease single vertex crease patterns. The crease pattern is parameterized by a, the smallest sector angle, and b, the sector angle adjacent to a, with the crease between a and b folding in the opposite direction from the other three creases. One can then choose the thickness t of the material to be produced and the largest fold angle r, here restricted to between 0° and 90°. The software draws construction lines for the offset width required by the fold angles. Then teeth are added. Slight modifications are made to trim volume that could intersect local to the vertex; see Figure 10. Through holes are added to each face. Holes, construction lines, and the volume trimmed faces can be hidden or shown independently. The drawing can then be downloaded as SVG and imported into a vector drawing program for printing on a 2-axis cutting tool.

[1] http://jasonku.mit.edu/four-crease-hinge-layout/

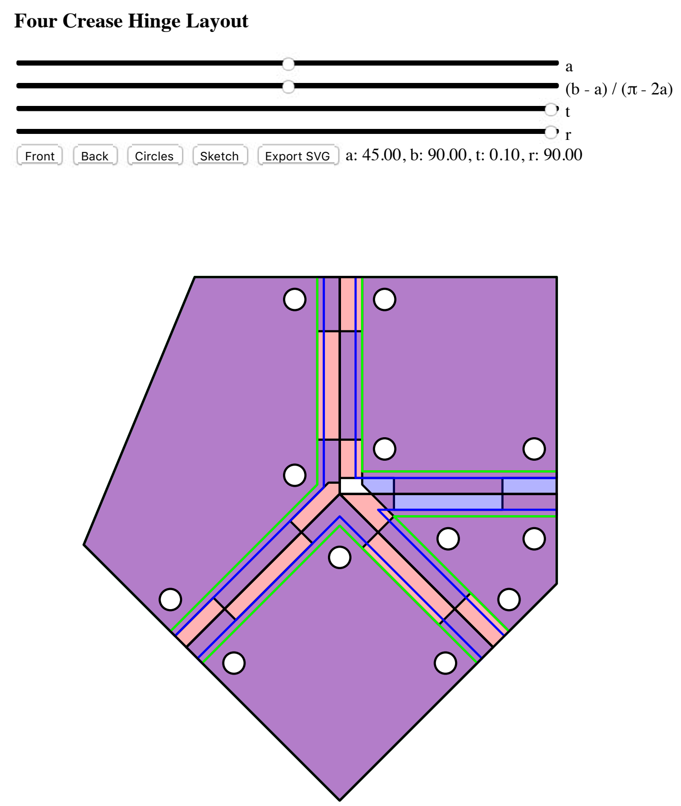

FIGURE 9. A web application generating cut patterns for flat-foldable four-crease single vertex crease patterns, allowing max fold angle 90°.

FABRICATION EXAMPLES

We used the software above to fabricate some single vertex mechanisms, which can be seen in Figure 11. The thick material is 3/16" clear acrylic with a folding layer made from 0.007" lightweight Tyvek fabric. The assembly is held together using press-fit unthreaded nylon spacers. Additionally, we fabricated some large angle hinges using both the piecewise constant and linearly varying strategies. These are shown in Figure 12.

We observed the linearly varying diagonal hinge to be more robust than the piecewise constant construction. The wide spacing between containing teeth in the piecewise constant model allowed non-negligible delamination to occur between the teeth. Additionally, the corners of the square cuts were susceptible to tearing under load on the hinge due to high concentration of stress at the hinge. The linearly varying model had substantially smaller spacing between containing tooth contacts, allowing less compliance. Additionally, the linearly varying model did not suffer from the same tearing weakness do to the geometry of the hole corners and the presence of more folding layer material at the hinge.

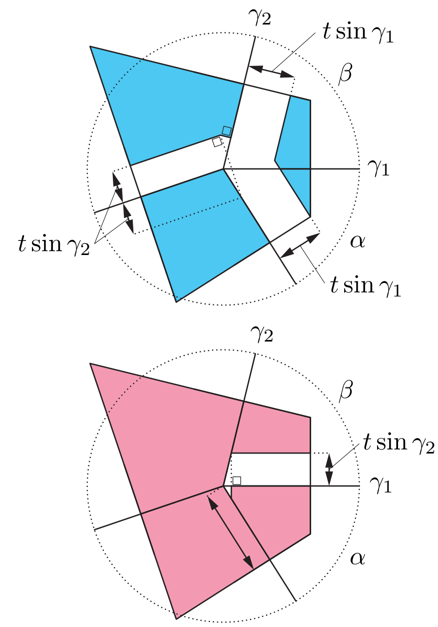

FIGURE 10. Details of additional volume removal to avoid material intersection local to a flat-foldable four-crease single vertex crease pattern.

Additionally, the techniques presented in this paper apply very generally to producing hinges using layered fabrication process. As such, these hinge making techniques can also be used in combination with the offset crease method, providing crease containment to two hinges, each with fold angle less than 90°. Figure 13 shows an offset crease model of a traditional bird base, using axially varying volume trimming to produce the hinges. The model was fabricated from water-jetted aluminum, fastened with screws.

CONCLUSION

In this paper, we have presented a general ideal hinge design, tailored for layered fabrication based on the volume trimming thickness compensation model. Volume trimming for both small and large fold angles can be adapted to retain thick material close to the fold line, allowing robust hinge construction. This hinge construction method can also readily be applied to the offset crease method for producing thick foldings with full range of motion to a facet parallel state. We imagine this hinge construction could be useful in other layered manufacturing approaches, for example in lithographic processes on the micro and nano scale. Because 2-axis cutting processes are readily accessible to many researchers, we hope that volume trimming will become more popular for fabricating thick folded structures, adapt-

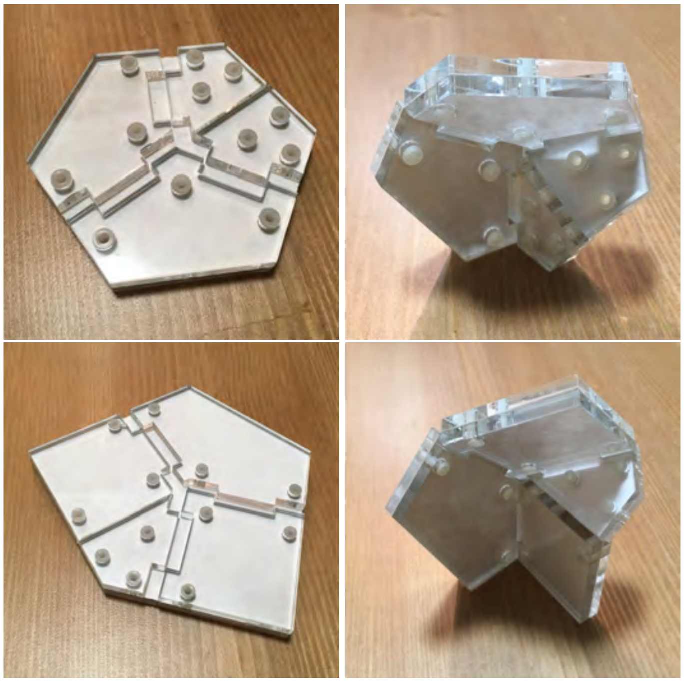

FIGURE 11. Two flat-foldable four-crease single vertex crease patterns made from acrylic.

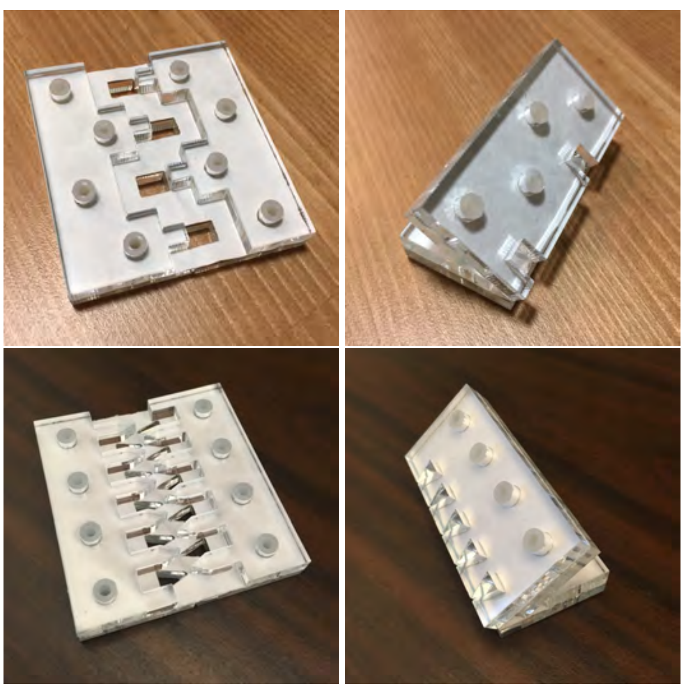

FIGURE 12. Hinges fabricated to demonstrate both piecewise constant [Top] and piecewise linear [Bottom] variation strategies for producing crease containment at large fold angles.

FIGURE 13. Axially varying volume trimming applied in combination with the offset crease method to produce a thick folding bird base from aluminum.

ing custom axial variation to fit a wide range of application.

In the future, we hope to combine this near-ideal hinge fabrication strategy presented in this paper, with bi-layer finite curvature active hinges, by changing hinge fabrication strategies along the length of a fold, to produce active hinges that also exhibit near-ideal hinge kinematics.

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