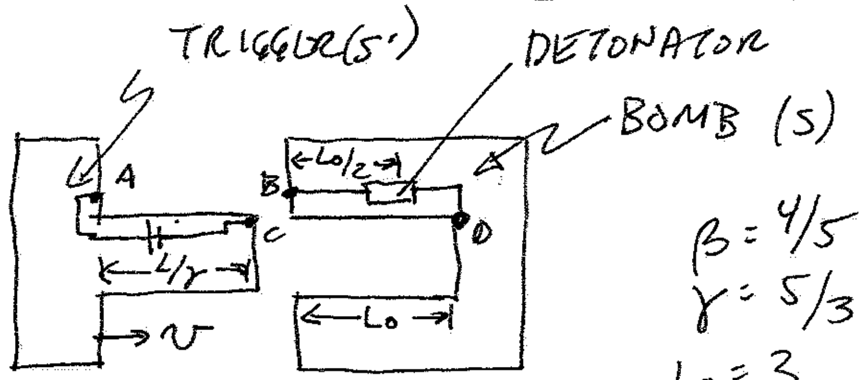
PRACTICE PROBLEMS

Oct 6, 2008 (1)

#1.

$L/\gamma = L_0$



$\beta = 4/5$
$\gamma = 5/3$
$L_0 = 3$

The bomb "assembles" by the trigger moving at velocity v into the bomb. A and B come into contact, as ~~do~~ do C and D. When ~~a~~ A and B contact, a signal propagates at velocity v^{4c} down the wire to the detonator. The same happens with C and D. The bomb channel depth is L_0 and the trigger has length $L_0 = L/\gamma$; L is the rest length of the trigger ~~S~~

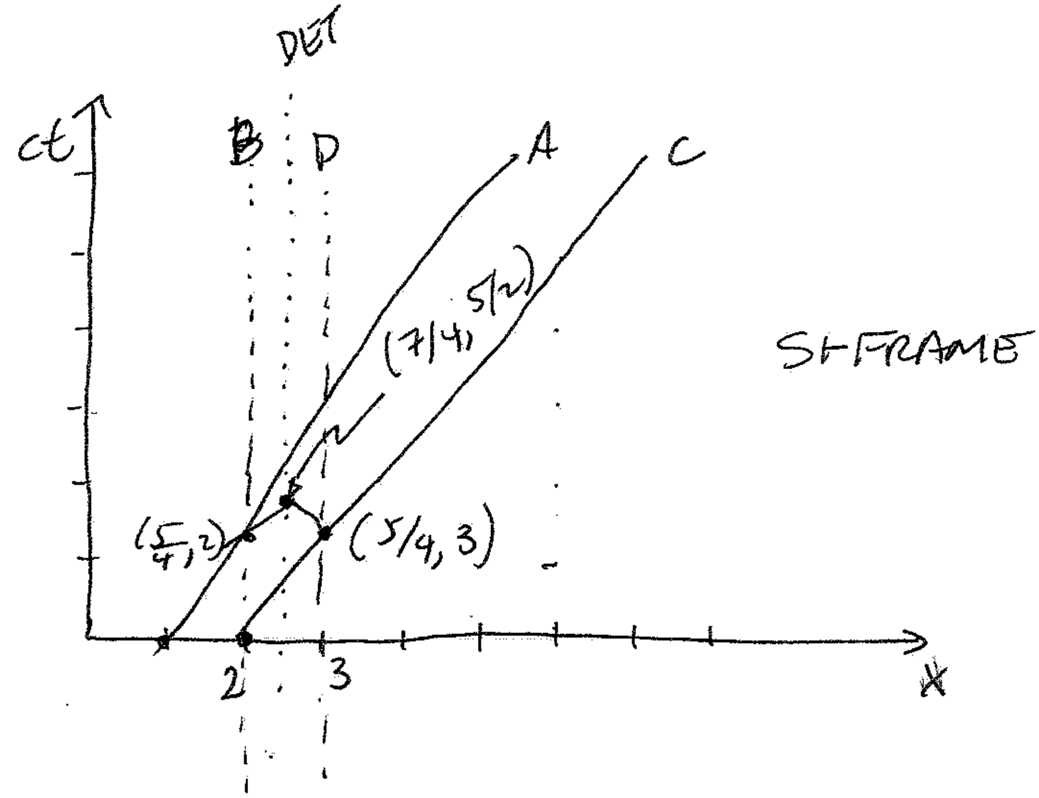
a) Draw a ~~set~~ Minkowski diagram for frame ~~S~~ S.

b) Draw the world lines for the contacts A–D and detonator. Draw the world lines for the ~~two~~ propagating signal.

c) Do the signals arrive at the detonator at the same time in S?

d) Add the (ct', x') axes and analyze the problem.

Solution #1



World line of C: $ct = \frac{5}{4}x + b = \frac{5}{4}x - \frac{5}{2}$

goes through $(0,2)$: $0 = \frac{5}{4} \times 2 + b = \frac{5}{2} + b$

$\rightarrow b = -5/2$

Intersects D at $x = 3m$,

$ct = \frac{5}{4} \times 3 - \frac{5}{2} = \frac{15}{4} - \frac{10}{4} = \frac{5}{4}$

World line of A: goes through $(0,1)$

$ct = \frac{5}{4}x + b \Rightarrow 0 = \frac{5}{4} + b \Rightarrow b = -\frac{5}{4}$

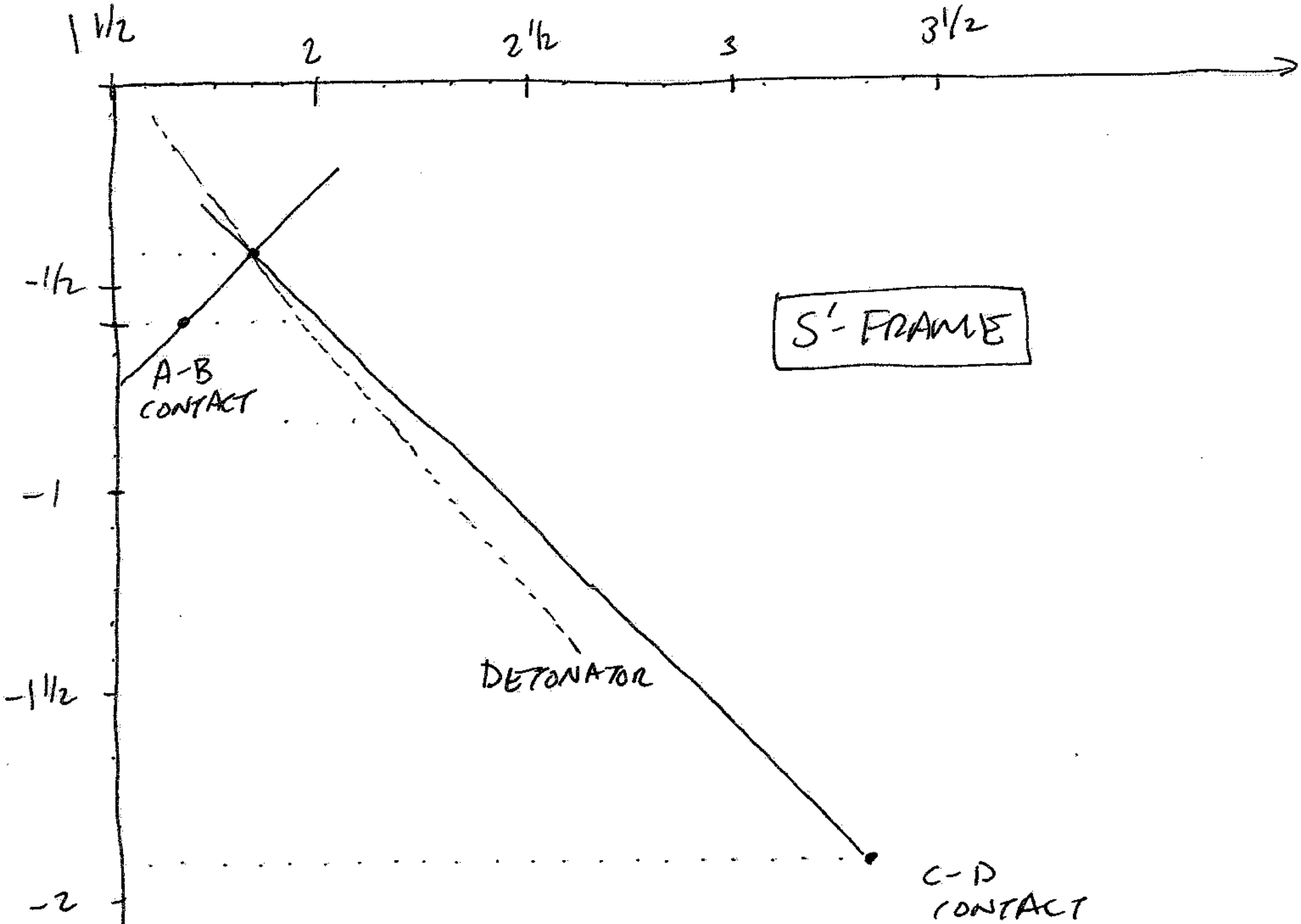
$ct = \frac{5}{4}x - \frac{5}{4}$

intersects C at $x = 2$, $ct = \frac{5}{4} \times 2 - \frac{5}{4} = \frac{10}{4} - \frac{5}{4} = \frac{5}{4}$

The signal from $(5/4, 2)$ must travel 1/2 to detonator at $x = 5/2$ ~~and arrives at 5/4~~ +

$ct = x + b \rightarrow 5/4 = 2 + b \rightarrow b = \frac{5}{4} - \frac{8}{4} = -3/4$

$ct = x - 3/4 \Rightarrow ct = \frac{5}{2} - \frac{3}{4} = \frac{10}{4} - \frac{3}{4} = \frac{7}{4}$



As you can see, in the S' frame, the detonator and signals from the contacts cross at the same space-time event.

We can check the interval

$$\cancel{S^2_{AB-DET} = \left(\frac{5}{2} - 2\right)^2 - \left(\frac{7}{4} - \frac{5}{4}\right)^2}$$

$$\cancel{= \left(\frac{5}{2} - \frac{4}{2}\right)^2 - \left(\frac{7}{4} - \frac{5}{4}\right)^2 = \left(\frac{1}{2}\right)^2 - \left(\frac{1}{2}\right)^2} = 0$$

$$\cancel{S^2_{CD-DET} = \left(\frac{5}{2} - 3\right)^2 - (}$$

So, there are three events

A − B contact at $(5/4, 2)$

C − D contact at $(5/4, 3)$

Arrival from A − B and C − D at $(7/4, 5/2)$

How do things look in S'?

$$\left.\begin{aligned} x &= \gamma x' + \beta\gamma c t' \\ ct &= \gamma c t' + \beta\gamma x' \end{aligned}\right\} \Rightarrow \begin{aligned} x' &= \gamma x - \beta\gamma c t \\ ct' &= \gamma c t - \beta\gamma x \end{aligned}$$

(sign negative because $x=0$ moves in the negative direction when viewed in S'

$S: (5/4, 2)$ in S' $\left(\frac{5}{3}\times\frac{5}{4} - \frac{4}{3}\times 2, \frac{5}{3}\times 2 - \frac{4}{3}\times\frac{5}{4}\right)$

A-B contact

$$\left(\frac{25}{12} - \frac{32}{12}, \frac{10}{3} - \frac{5}{3}\right)$$

$$\boxed{\left(-\frac{7}{12}, \frac{5}{3}\right)}$$

$S: (5/4, 3)$ in S' $\left(\frac{5}{3}\times\frac{5}{4} - \frac{4}{3}\times 3, \frac{5}{3}\times 3 - \frac{4}{3}\times\frac{5}{4}\right)$

$$\left(\frac{25}{12} - \frac{48}{12}, \frac{15}{3} - \frac{5}{3}\right) = \boxed{\left(-\frac{23}{12}, \frac{10}{3}\right)}$$

$S: (7/4, 5/2)$ in S' $\left(\frac{5}{3}\times\frac{7}{4} - \frac{4}{3}\frac{5}{2}, \frac{5}{3}\times\frac{5}{2} - \frac{4}{3}\times\frac{7}{4}\right)$

$$\left(\frac{35}{12} - \frac{40}{12}, \frac{25}{6} - \cancel{\frac{28}{12}}\frac{14}{6}\right) = \boxed{\left(-\frac{5}{12}, 11/6\right)}$$

$$S^2_{AB\text{-}DET} = \left(-\frac{7}{12} - \left(-\frac{5}{12}\right)\right)^2 - \left(\frac{5}{3} - \frac{11}{6}\right)^2$$

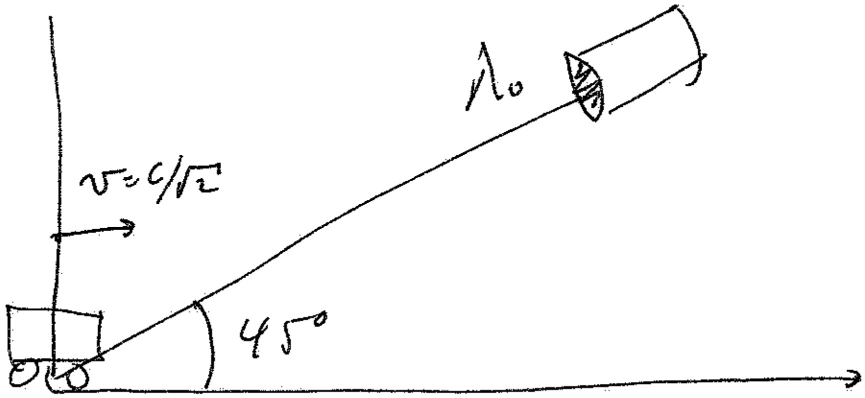
$$= \left(-\frac{1}{6}\right)^2 - \left(-\frac{1}{6}\right)^2 = 0$$

$$S^2_{CD\text{-}DET} = \left(-\frac{23}{12} - \left(-\frac{5}{12}\right)\right)^2 - \left(\frac{10}{3} - \frac{11}{6}\right)^2$$

$$= \left(-\frac{18}{12}\right)^2 - \left(\frac{9}{6}\right)^2 = \left(-\frac{3}{2}\right)^2 - \left(\frac{3}{2}\right)^2$$

So, in S', the photons traveled very different distances, but this is compensated by one starting much earlier than the other.

#2



a flash light with area A in rest frame is viewed by a Cam moving at $v = c/\sqrt{2}$. Angles as shown.

a) For λ observed.

b) Find ~~dist~~ angle observed by Cam.

c) Find shape and size of ~~the~~ spot.

a) $\lambda = \gamma \lambda_0 (1 - \beta \cos\theta)$

$$\gamma = \frac{1}{\sqrt{1 - 1/2}} = \frac{1}{\sqrt{1/2}} = \sqrt{2}$$

$$\lambda = \sqrt{2}\,\lambda_0\left(1 - \frac{1}{\sqrt{2}}\frac{1}{\sqrt{2}}\right) = \frac{\sqrt{2}}{2}\,\lambda_0 = \frac{\lambda_0}{\sqrt{2}}$$

b) $u_x = c/\sqrt{2}$

$u_y = c/\sqrt{2}$

$$u_x' = \frac{u_x + v}{1 + v u_x/c^2} = \frac{2/\sqrt{2}}{1 + 1/2} = \frac{\sqrt{2}}{3/2} = \frac{2\sqrt{2}}{3}$$

$$u_y' = \frac{u_y}{\gamma(1 + v u_x/c^2)} = \frac{1/\sqrt{2}}{\sqrt{2}\,(3/2)} = \frac{2}{2\sqrt{3}} = \frac{1}{\sqrt{3}}$$

$$\tan\theta' = \frac{u_y'}{u_x'} = \frac{1/\sqrt{3}}{\frac{2\sqrt{2}}{3}} = \frac{\sqrt{3}}{2\sqrt{2}}$$

c) $A' = \frac{A}{\gamma} = \frac{A}{\sqrt{2}}$

#3. An artillery shell goes up then down

$$x(t) = -\frac{1}{2} g t^2 + v_0 t$$

a) Find time in lab frame

b) Find time in shell rest frame.

a) $x(\tau) = 0 = \tau\left(-\frac{1}{2} g \tau + v_0\right)$

$\tau = 0$ at start

$$-\frac{1}{2} g \tau + v_0 = 0 \Rightarrow \frac{1}{2} g \tau = v_0 \Rightarrow \boxed{\tau = \frac{2 v_0}{g}}$$

b) In rest frame $c\,dt' = ds$

$$ds = \sqrt{1 - v^2/c^2}\, dt \rightarrow \tau' = \int_0^{\tau} dt \sqrt{1 - v^2/c^2}$$

$v(t) = -gt + v_0$

~~Complete the squares.~~

Take $u = (gt + v_0)/c^2$

$du = -g\,dt/c^2$

$$s = \tau' = \int_{-v_0/c^2}^{(-g\tau+v_0)/c^2} \left(\frac{-c^2}{g}\right) du \sqrt{1-u^2}$$

$u = \sin x$

$du = \cos x \, dx$

$$\tau' = \int_{\sin^{-1}(-v_0/c^2)}^{\sin^{-1}(-g\tau+v_0)/c^2} \cos x \, dx \sqrt{1-\sin^2 x}$$

$$= \int \cos^2 x \, dx = \int \left(\frac{1}{2} + \frac{1}{2} \cos 2x\right) dx$$

$$= \frac{x}{2} + \frac{1}{4} \sin 2x \Big|_{x=\sin^{-1}(-v_0/c^2)}^{x=\sin^{-1}(-g\tau+v_0)/c^2}$$

Oh, not so easy.